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Density current flux due to bubble plume using a new air energy system

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ABSTRACT

The Dam Air-energy System (DAS) is a new energy system using compressed air with high efficiency in dams. Several countermeasure facilities were installed in the Haneji Dam for water quality improvement as the first full-scale system using DAS. DAS facilitates artificial circulation of water by using a bubble plume caused by the production of compressed air by the DAS. Due to the efficiency of DAS, a large amount of compressed air (9.7 m3/min) can create a bubble plume. Field experiments were conducted to elucidate the flux in the horizontal density currents caused by the bubble plume. Based on temporal changes in the vertical profiles of water temperatures, flow rates of surface currents and intrusion as the middle layer flow were estimated. This method is applicable to the Haneji Dam due to the long residence time of water as hydrologic characteristics of the Haneji Dam reservoir. It was observed that a large amount of current flux was produced by the DAS-bubble plume system. By comparison of the experimental results with many previous studies, empirical equations for estimating the flux in current due to the bubble plume are proposed.

1. INTRODUCTION

One important problem in the management of dam reservoirs is the deterioration of water quality due to eutrophication by cyanobacterial bloom. As one useful countermeasure to these problems, artificial circulation has been applied in various countries around the world (Cooke et al. 2005). Although much scientific and technological research and development related to artificial circulation has been carried out so far, application guidelines based on the mechanisms of various effects of this measure have not yet been developed. The mechanism of this measure consists of complicated processes of responses of the phytoplankton community in addition to habitat conditions caused by density flow due to a bubble plume and vertical mixing by natural external force. This study aims at quantitative evaluation of the density flow rate occurring in a wide area due to a local bubble plume, which is important among the processes. In recent years, it has been reported that these trends are different from the results of previous research on stratified flow in the flow rate of horizontal density generated by a bubble plume in actual reservoir conditions (Furusato et al. 2015, Furusato & Inuyama 2016). However, because these results are based on results obtained under limited conditions, further verification based on field observation data is necessary.

A dam air-energy system (DAS) was introduced as a facility at the Haneji Dam in the northern part of the main island of Okinawa (Kawasaki 2006). A part of the compressed air produced is used as a bubble circulation countermeasure for water quality preservation. The purpose of this research was to evaluate the rate of horizontal flow over a wide area as basic information to construct a design for management of bubble circulation. For this purpose, this paper reports the field survey results of deformation of water temperature stratification due to an air bubble circulation facility utilizing DAS in the Haneji Dam, and analyze it together with other previous research results.

2. OUTLINE OF DAS AND HANEJI DAM

2.1 DAS

The DAS is a clean energy system that not only lowers dam management costs, but also to resolves environmental problems by improvement of water quality in reservoirs and reduces CO_2 by unused clean hydraulic power.

The system directly produces compressed air without first producing electric power and utilizes compressed air for various purposes, realizing low cost and high performance by its simple configuration of equipment. A full-scale DAS is used at the Haneji Dam, and it makes a very significant contribution to both the economy and conservation of the environment.

2.1.1 Basic Outline of DAS

The basic DAS flow is, “compressing air → storing the compressed air (can be omitted) → using the compressed air.” The direct use of compressed air produced by clean energy surrounding a dam as clean energy to power aeration, pumping up, cooling, and other management facilities functions that consume large quantities of energy can prevent great environmental damage and provide economic benefits.

Air is an energy medium with three functions: environmental (aeration, supplying oxygen, etc.), temperature (expansion, cooling, etc.), and power (pumping up, spraying, etc.). In the past, energy created by dams was assumed to mean only electric power, but the development of DAS will result in multiple forms of energy.

2.1.2 Economic Characteristics of DAS

Many dams incur high management costs for aeration, pumping up, fountain operation, and cooling and heating, so to lower management costs in the future, it will be important to use hydraulic power and other nearby power sources. However, at medium-sized and small dams, it is often impossible to produce hydroelectric power because the cost of production is not covered by the income it earns. Because the configuration and structure of DAS equipment are simple, it is much more economical than a hydroelectric system.

Specifically, it sharply reduces equipment costs—(1) the cost of producing air is 1/3 of the cost of producing electricity and (2) the cost of spraying is 1/10 of what it was formerly. At the same time, it improves reliability by simplifying the equipment. Maintenance costs are also sharply reduced because the source energy is free. These benefits greatly reduce the dam's life cycle costs (Kawasaki 2006).

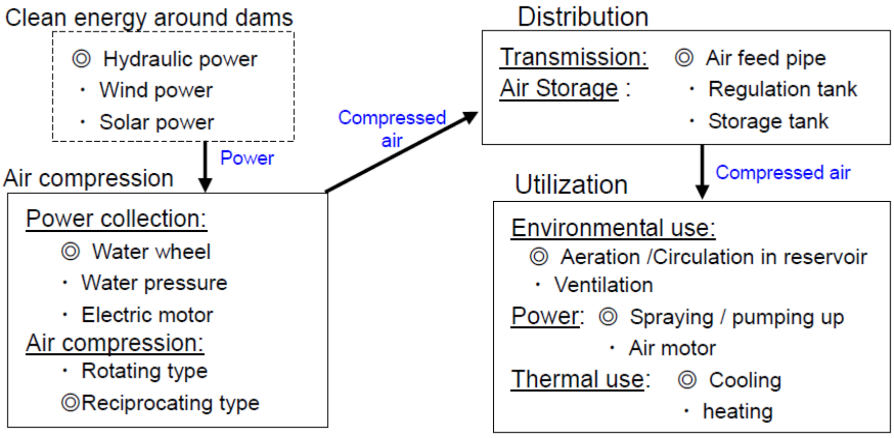
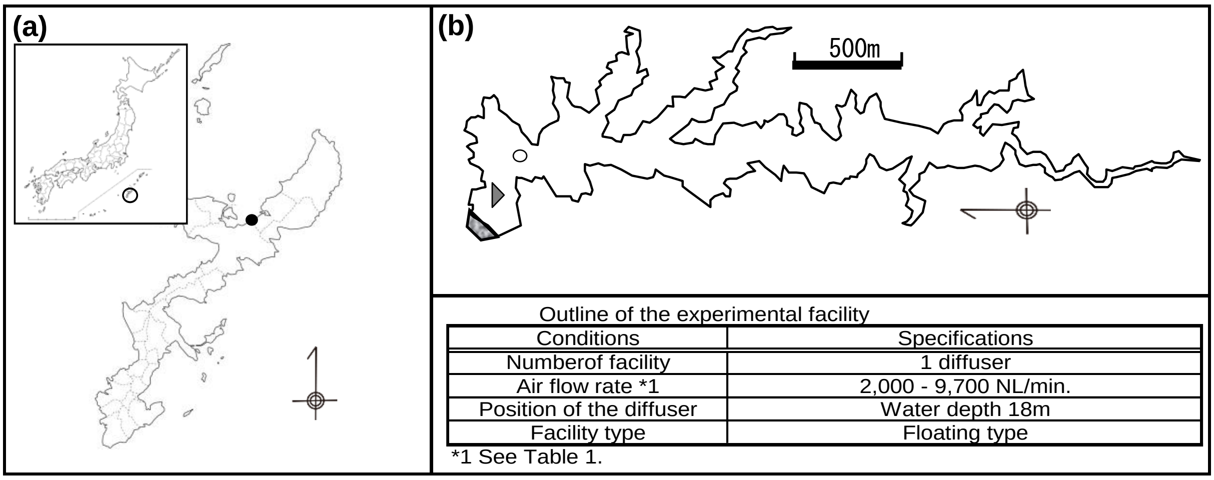


Figure 1. Configuration of DAS (◎: adopted at Haneji Dam)

2.2 Haneji Dam

This research was conducted in the Haneji Dam reservoir located in Nago City, in the northern part of the main island of Okinawa. The Haneji Dam is a central core rock-fill dam (height 66.5 m, width 198.0 m) constructed at 3.1 km from the East China Sea (catchment area 14.2 km^2, length of the river 12.3 km) of the secondary river Haneji-ohkawa river (Figure 2). The reservoir is named Saion-Akeomi Lake and classified as a large stagnant water body in Okinawa Prefecture (total volume 19.8 million m^3, surface area 1.2 km^2). On the other hand, the catchment area is 10.9 km^2, which is less than 10 times the reservoir surface area, and the catchment area is small relative to the reservoir scale as an ordinary multipurpose dam reservoir in Japan.



Outline of the experimental facility

Conditions	Specifications
Numberof facility	1 diffuser
Air flow rate *1	2,000 - 9,700 NL/min.
Position of the diffuser	Water depth 18m
Facility type	Floating type

*1 See Table 1.

Figure 2. Location of Haneji Dam and details of air diffuser

The position of the diffuser is a dam site (Figure 2(b)). Table 1 shows the experimental conditions. In this dam reservoir, hypolimnetic oxygenation system_as another countermeasure device. Deep aeration facilities to supply Dissolved Oxygen(DO) to the hypolimnion have also been introduced in Haneji Dam (Kawasaki 2006). The facility was operated at 5,000 NL/min. until the beginning of August 2004 before the start of the bubble circulation experiment, the second year of the experiment, and then at 2,000 NL/min. until late September 2004. As a result, the DO concentration was kept above a certain level mainly at the suction port elevation (EL. 30.5 m) and ejection altitude (EL. 40.5 m), and the anaerobic hypolimnion was suppressed. (Kawasaki 2006). However, this study does not aim to evaluate the effect of the deep aeration facilities because the purpose of this paper is to report basic scientific information about bubble circulation countermeasures.

3. MATERIALS AND METHODS

3.1 Parameters

3.1.1 Stratified flow and bubble plume

Several parameters related to the characteristics of the bubble plume structure and stratified flow are used in this research.

$$N = \sqrt{\frac{g}{\rho}\frac{d\rho}{dz}} \tag{1}$$

$$H_{\mathrm{R}} = \frac{H}{H + H_A} = \frac{H}{H_T} \tag{2}$$

$$P_{\mathrm{N}} = \frac{N^3 H^4}{g Q_B} \tag{3}$$

$$M_{\mathrm{H}} = \frac{Q_B g}{4\pi\alpha^2 {u_s}^3 H} \tag{4}$$

$$Q_S = \left[\frac{(Q_B g)^3}{N^5}\right]^{\frac{1}{4}} \tag{5}$$

$$l_O = \left(\frac{Q_B g}{N^3}\right)^{\frac{1}{4}} \tag{6}$$

Where N: buoyancy frequency (s^{-1}): H_R: dimensionless air discharge depth with respect to the bubble plume structure, P_N : Plume number, M_H : non-dimensional parameter corresponding to the ratio of the bubble buoyancy added to the rising water flow rate, Q_s : the scale of the rising flow rate of the bubble jet, l_o: Ozmidov length, g: gravitational acceleration (m/s^2), ρ: density of water (kg/m^3), z: depth (m), H : air diffuser depth (m), H_A : atmospheric pressure head (～10.2 m), H_T : total head (m) ($H_T = H + H_A$), Q_B : air flow rate under air diffuser depth pressure (m^3/s) ($Q_B = Q_a H_A / H_T$), Q_a : air flow rate under atmospheric pressure (Nm^3/s), α: entrainment coefficient (=0.083 (Milgram 1983)), and u_s : slip velocity (～0.3 m/s (Kobus 1968)).

In this paper, from the consistency of the linear density stratification assumption in the plume number, the density gradient was calculated by the ratio of the density difference between surface and air diffuser depth and air diffuser depth. The density of water is calculated by water temperature.

N is one of the fundamental parameters in stratified flow. The characteristics of a bubble plume are described by three dimensionless numbers (H_R,P_N,M_H) ((2), (3), (4) (Lemckert & Imberger 1993, Asaeda & Imberger 1993). H_R is expressed by the ratio of the bubble discharge water depth to the total water head (atmospheric pressure + water pressure). P_N is non-dimensional number for the ratio of density stratification and buoyancy of the bubbles (Asaeda & Imberger 1993). M_H is used by some previous studies (McDougall 1978, Asaeda & Imberger 1993, Lemckert & Imberger 1993). Q_s is used for dimensionless sizing of the wide area of horizontal density flow (Lemckert & Imberger 1993, Asaeda & Imberger 1993, Socolofsky & Adams 2005). l_0 is the length scale as another basic physical quantity (Ozmidov 1965). As can be inferred from (3) and (6), P_N corresponds to the fourth power of the ratio of H and l_0 (Socolofsky & Adams 2005).

3.1.2 Structure of bubble plume

The basic structures of bubble plumes can be classified into three types depending on the P_N (Asaeda & Imberger 1993).In the case where the buoyancy of the discharge bubble is sufficiently large compared to the stratification strength of the surrounding water, at P_N <500, the bottom layer water having a high density near the discharge port is lifted to the water surface. It spreads to the surroundings and plunges at a fixed distance. In the case where the buoyancy of the discharge bubble is sufficiently large compared to the stratification strength of the surrounding water (PN < 500), the bottom layer water having a high density near the discharge port is lifted to the surface. After this plume reaches the surface, it spreads to the surroundings and submerges at a fixed distance(Figure 3).On the other hand, in a large P_N condition (P_N > 500), different bubble plume structures are formed. P_N is small in actual reservoirs (Kranenburg 1979) bacause density stratification is caused only by the difference in water temperature, in addition to the fact that the amount of air discharged is extremely large compared to indoor experiments. Furusato et al. (2015) and Furusato & Inuyama (2016) pointed out that the P_N of many previous studies (Asaeda & Imberger 1993, Lemckert & Imberger 1993, Topham1975, Matsunashi & Miyanaga 1990) ranged from 101 to 104. Recently, wide area horizontal flow rates under small P_N (<1) based on practical reservoirs conditions were evaluated (Furusato et al. 2015, Furusato & Inuyama 2016).

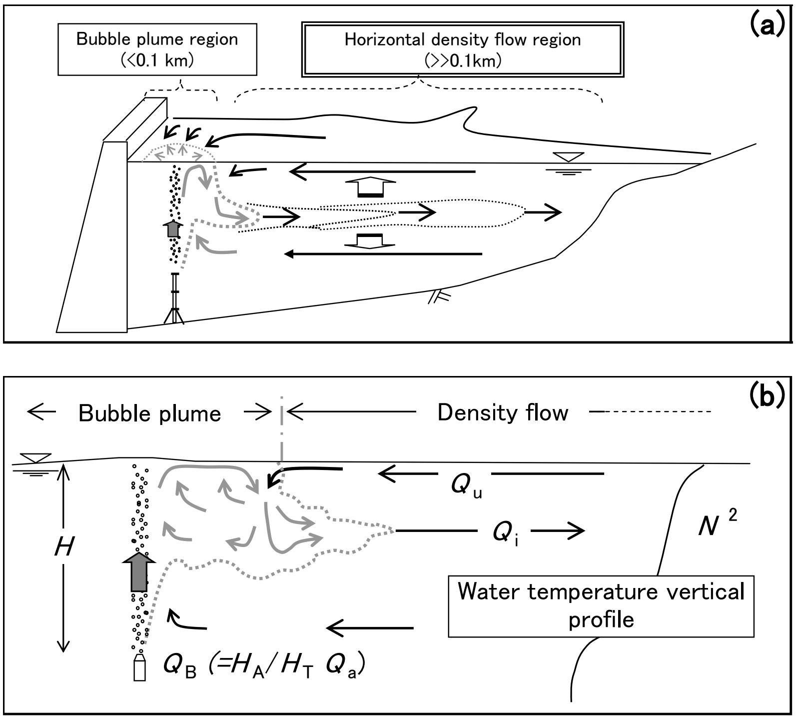


Figure 3. Bubble plume and wide area horizontal density flows

3.1.3 Facility scale of air diffusers

The most important condition for design of an artificial circulation facility is the method to determine the facility scale, such as the air flow rate and the number of air diffusers (Gotou et al. 2008).

$$k = \frac{\sum \sqrt{Q_{0N}}}{A} \tag{7}$$

where k is facility scale for artificial circulation devices with respect to water body size, Q_{0N} is air flow rate under atmospheric pressure as practical unit (NL/min.), and A, reservoir surface area (km^2).

These parameters have been used in Japan in the last decade for design of artificial circulation countermeasures using a bubble plume for preventing eutrophication. The range of appropriate *k* is from 200 to 300 as empirical values (Gotou et al. 2008).

3.1.4 Flow rate of wide area horizontal density flow

The flow rate of the horizontal density flow generated by the bubble plume was estimated by the following method based on the continuous equation according to previous studies (Asaeda & Imberger 1993, Lemckert & Imberger 1993, Zic et al. 1992, Furusato & Inuyama 2016).

$$Q_x = \frac{|\Delta V_x|}{\Delta t_x} \tag{8}$$

where *ΔV* is the amount of change in the surface and middle layer volumes during *Δt*, *Δt* is the time interval for *Q* calculation, x is the index number of the target density flow, x = *u* is the surface layer flow, x = *i* is the intrusion In the middle stream. *ΔV* was determined from changes of the water temperature stratification form caused by horizontal density flow generated by the bubble plume. The change volume was evaluated from the elevation - volume data of the dam reservoir. The absolute value of the numerator on the right side of (8) is taken into consideration in the case of the surface layer flow. The decrease in the surface layer volume corresponds to the surface horizontal flow rate.

Since the turnover rate at the Haneji Dam reservoir is relatively small, the influence of the inflow and discharge on the stratification form is neglected. In addition, *ΔV* was evaluated after confirming that there was no water temperature stratification deformation due to wind stress as another external mixing force or mixing by water surface cooling using the weather and flow condition data of the experiment period.

3.2 Field experiments and flux in density currents due to bubble plume using compressed air produced by DAS

In the field survey, vertical distributions (water depth 1m pitch) of water temperature and DO (1 m depth pitch) were measured approximately every week from a boat using a multi-parameter water quality meter (TOA - DKK, portable multi-item water quality meter WQC - 24) (Figure 1(b)). Although not shown in this paper, the vertical distribution of water temperature and DO in the horizontal direction were uniform in each survey (Kawasaki 2006), so only the data of dam site were used in this paper.

Table 1. Experimental cases

CASE*1	Period *2	Air flow rate*3
03-1*4	10 July - 23 July (9 July - 16 July)	2,000
03-2*5	23 July - 6 Aug (23 July - 30 July)	2,000
03-3	13 Aug - 24 Aug (13 Aug - 20 Aug)	9,700
04-1*6	28 July - 16 Aug (21 July - 6 Aug)	9,700
04-2	26 Aug - 25 Sep (25 Aug - 3 Sep)	8,000

*1 : 03 - indicates 2003, 04 indicates 2004.

*2 : In parentheses, the evaluation period of horizontal flow rate (water temperature survey date)

*3 : Units, NL/min.

*4 : In this case only, the discharge air size was microbubbles (bubble diameter 0.4 mm), and in the other cases compressed air was discharged from a single hole of a discharge port with a diameter of 40 mm.

*5 : For this case only the horizontal flow rate was estimated from both water temperature and DO.

*6：21 July data was used because the survey was not carried out on 28 July. However, since the change in the water temperature vertical distribution from 21 July to 28 July was negligible, *Δt* on 28 July was assumed to be the initial value. This also applies to CASE 04-2, and 25 Aug data is used as an initial value, but in *Δt* it is treated as 26 Aug data.

4. RESULTS AND DISCUSSIONS

4.1 Facility scale

Figure 4 shows the facility scale of the air diffuser in Haneji Dam compared to other dam *k* values (JWA 2006). The *k* value of this experiment conducted in Haneji Dam was about 80. The facility scale of this experiment at the Haneji Dam was small compared to many other Japanese reservoirs (JWA 2006). Therefore, complete mixing, in particular surface diurnal thermoline, will be difficult at the experimental facility at the Hanaji Dam. However, the purpose of this experiment is to elucidate the flow rate that produced a bubble plume. In such a situation, the remaining surface thermocline makes the estimation easy.

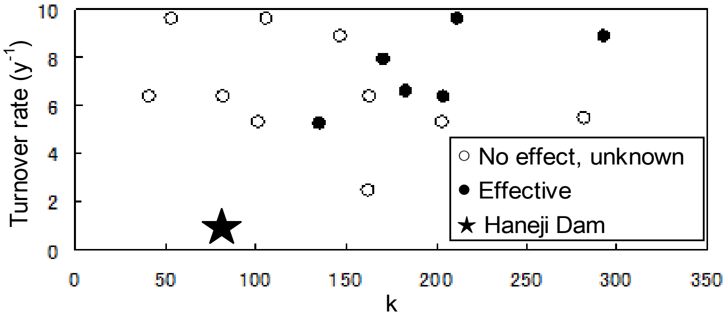


Figure 4. Comparison of facility scale value (*k*) of this experiment with other dam reservoirs in Japan

4.2 External forces during the experiments

Figure 5 shows the temporal changes of weather and flow conditions during the four months from June to September, including the field experiment period implemented during the two years (2003 and 2004). External forces of the weather parameters for vertical mixing were used as basic information for setting the evaluation period of the horizontal flow rate based on the field experiment.

In both years, we selected a survey date that avoided the effects of deformation of water temperature stratification due to weather and river inflow and determined the investigation day to be used for flow rate evaluation of horizontal density flow as shown in Table 2.

4.3 Vertical profiles of water temperature and DO

Figure 6 shows the temporal changes in the vertical distribution of water temperature and DO during the experiment. Data is shown by selecting the start and end of each experimental case (Table 1, Figure3). In both years, the vertical distribution of water temperature and DO was deformed by the horizontal density flow generated by the bubble plume, and toward the bubble diffuser height (indicated by an inverted triangle in the Figures), the tendency for the mixed layer to enlarge from the middle layer was remarkable. The vertical distribution of DO in 2004 was the result of the operation of the deep aeration facility described above, and the DO is high in the deep part (deeper than Elevation 48 m). Because the operation of the deep aeration facility in 2004 stopped with the start of this experiment, the DO concentration in the hypolimnion gradually decreased after 28 July, the start of the experiment. Based on these conditions, it was decided to evaluate the wide horizontal flow rate generated by the bubble circulation from the deformation of the water temperature and the vertical distribution of DO. The horizontal flow rates estimated by the water temperature stratification deformation method are shown in Table 2 using the water temperature data of the evaluation period selected.

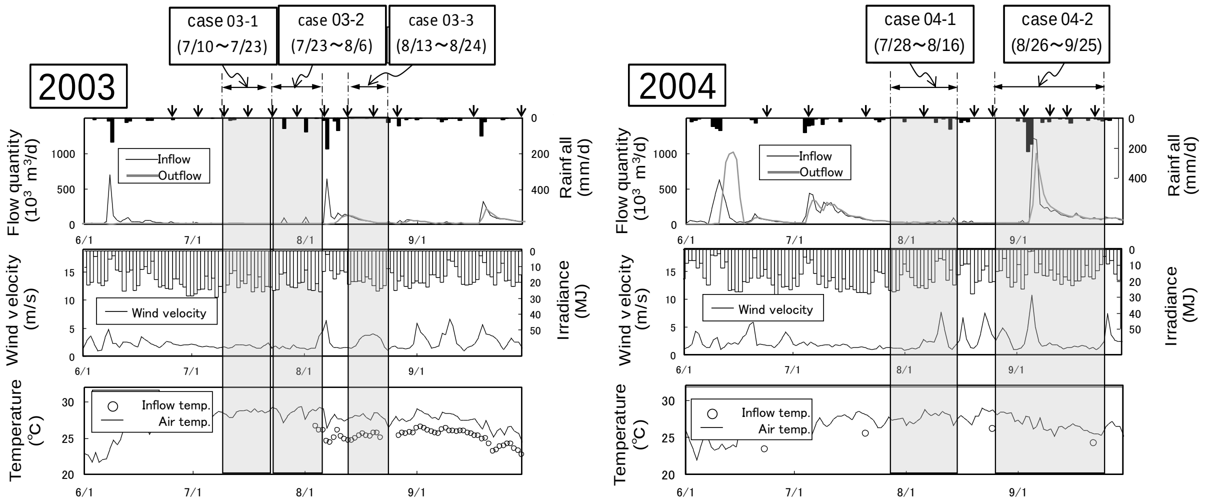


Figure 5. Temporal changes of weather and flow conditions during the field experiment period

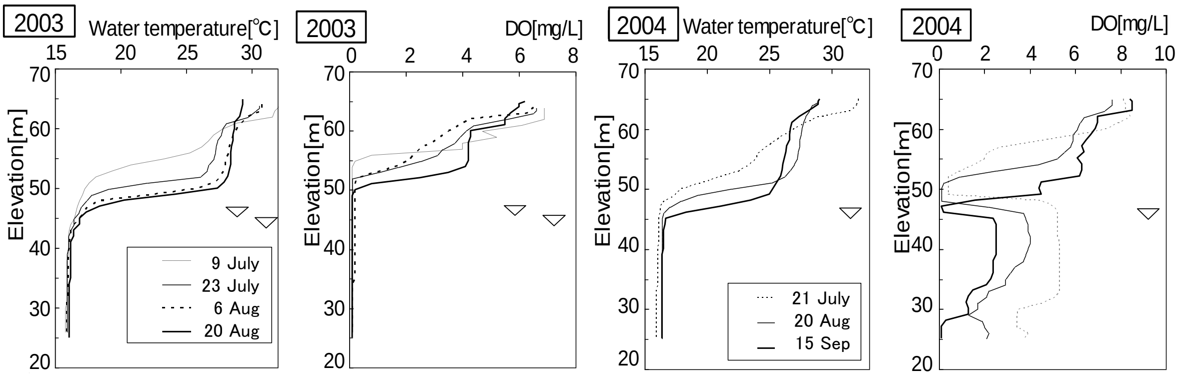


Figure 6. Temporal changes in vertical distribution of water temperature and DO during field experiments (∇ in the figure indicates the air diffuser depth (18 m)). As the water level rose during the experiment in 2003, water depths of 9 July and 20 Aug are shown as the positions corresponding to the first and last data shown.)

4.4 Bubble plume characteristics of experiments

Figure 7(a) shows the parameters (P_N, M_H) of these experiments related to the bubble plume form compared with the previous studies (Asaeda & Imberger 1993, Lemckert & Imberger 1993). In the figure, the three categories of bubble plume structures are shown based on a previous study (Asaeda & Imberger 1993). As in the previous study (Lemckert & Imberger 1993), which had similar arrangements, a negative relationship was confirmed for both indices. The conditions of the bubble plume of this research were intermediate conditions between these previous research examples and the recent research examples of the authors (Furusato et al. 2015, Furusato & Inuyama 2016).

4.5 Relationships between P_N and horizontal density flow rates

The field observations of the horizontal flow rate shown in Table 2 are rendered dimensionless by Q_S and show the relationship with P_N (Fig. 7). In both the dimensionless surface layer flow rate (Q_u/Q_S) (Fig. 7 (b))) and the middle layer flow rate (Q_u/Q_S) (Fig. 7(c)), the tendency of the low P_N number condition to decrease, which has been reported in recent years (Furusato et al. 2015, Furusato & Inuyama 2016), was confirmed.

From these data, we obtained the approximate equations of the surface layer and middle layer flow rate as a function of P_N like that of the previous studies (Asaeda & Imberger 1993, Lemckert & Imberger 1993, Furusato et al. 2015, Furusato & Inuyama 2016) ((9)–(12)). For an approximation, P_N

<100 data was targeted. In addition to the tendency in Figure. 7, this was set as a value (Socolofsky & Adams 2005) that corresponds to $P_N{}^{1/4}$ (= H / l_O), corresponding to about 3, as the condition in which the bubble jet reaches the water surface sufficiently. The boundary values of P_N in (9)-(12) are the intersection point of approximate lines in the surface layer flow and the middle layer flow, respectively.

Table 2 . Experimental conditions and field survey results

case	N^2 s^{-2}	Q_a m^3/s	P_N -	Q_s m^3/s	Q_u m^3/s	Q_i m^3/s
03-1	0.00175	0.033	64.0	10.77	3.00	5.96
03-2W	0.00169	0.033	60.9	11.00	0.54	3.02
03-2D	0.00169	0.162	12.6	35.94	2.34	2.89
03-3	0.00120	0.162	7.6	44.41	3.50	7.97
04-1	0.00121	0.162	7.6	44.19	2.20	6.79
04-2	0.00071	0.133	4.2	53.24	5.02	6.80

Surface layer flow (Q_u);

$$Q_u = 0.122 Q_s \qquad P_N > 7.2 \tag{9}$$

$$Q_u = 0.0375 Q_s P_N{}^{0.598} \qquad P_N < 7.2 \tag{10}$$

Middle layer flow (Q_i);

$$Q_i = 0.68 Q_s \qquad P_N > 65.8 \tag{11}$$

$$Q_i = 0.0736 Q_s P_N{}^{0.531} \qquad P_N < 65.8 \tag{12}$$

In previous studies (Asaeda & Imberger 1993, Lemckert & Imberger 1993), the non-dimensionalized Q_u and Q_i values by Q_S were constants with respect to P_N. On the other hand, recent studies (Furusato et al. 2015, Furusato & Inuyama 2016) reported that in the practical condition, P_N in the actual dam reservoirs, these non-dimensionalized values tend to decline. The data obtained in this study were consistent with these recent research results. As Furusato et al. (2015) pointed out, it is presumed that such a relationship is caused by a decrease in the amount of ambient water entrainment at the outer edge of the bubble plume region under the low P_N condition. However, since the detailed hydraulic mechanism of this phenomenon is unknown, study of the mechanism under low P_N condition it is required in the future.

5. CONCLUSION

Field observation of horizontal density flow caused by bubble circulation using compressed air produced by DAS was performed in a dam reservoir dam reservoir. The flow rate of horizontal flow was evaluated by water temperature stratification deformation method. Field observations were conducted during the initial impoundment period, which tended to have a long residence time. From this weather condition and flow regime, this period was regarded as a condition with little influence of factors influencing water temperature stratification deformation other than bubble circulation. As a result, it was observed that the maximum surface horizontal flow was about 5 m³/s, and the middle layer density flow (intrusion) was about 8 m³/s. The condition of the bubble plume in this study was compared with the previous knowledge using P_N, and it corresponded to the intermediate condition of the previous studies. As in recent research (Furusato et al. 2015, Furusato and Inuyama 2016), it was confirmed that the dimensionless horizontal density flow rate decreases according to the decrease of P_N under the condition of a certain P_N or less. Based on these results, empirical equations for evaluating the flow rate of horizontal density flow due to bubble circulation were proposed.

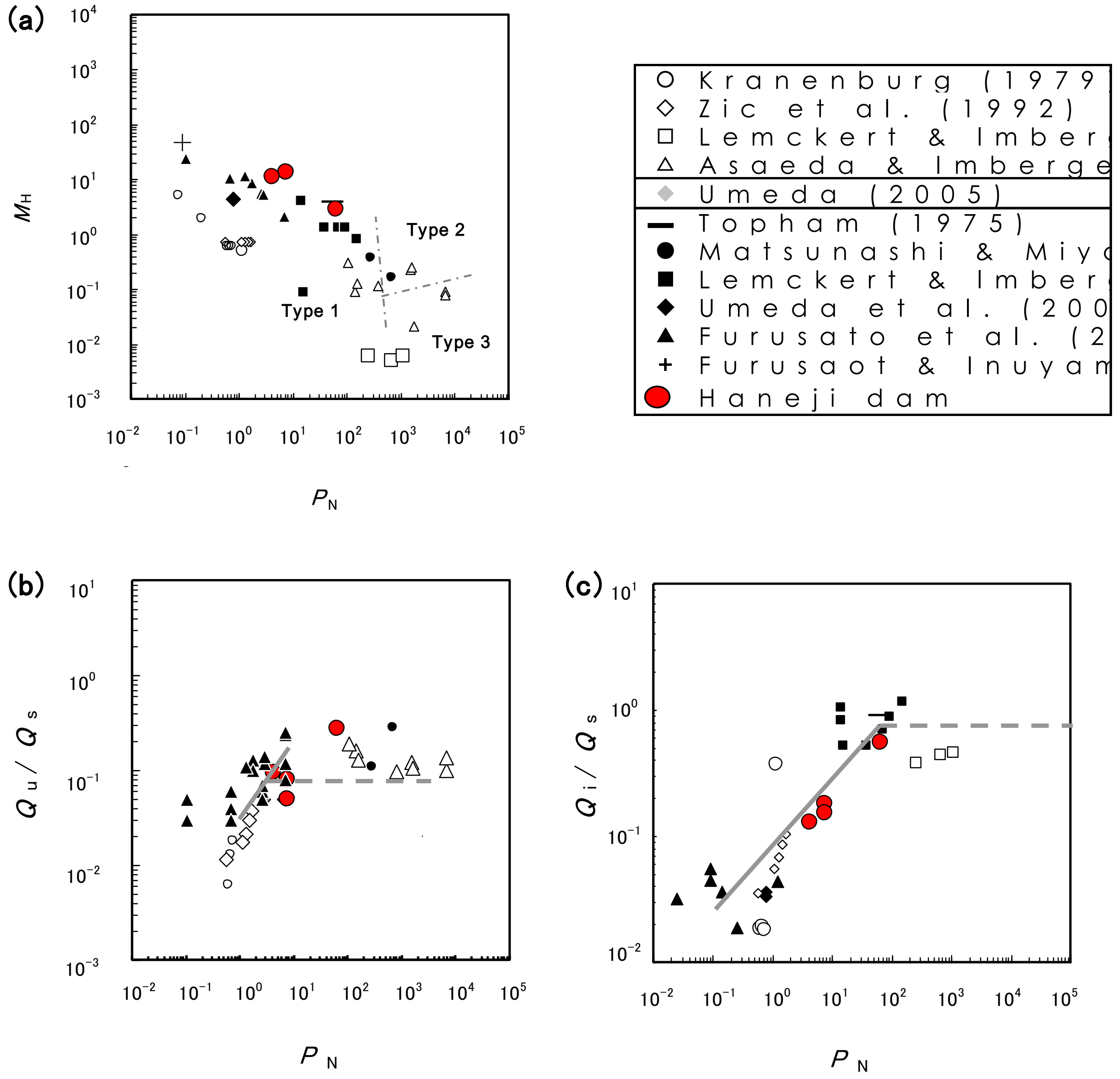


Figure 7 . The relationships between P_N and M_H(a), non-dimensional Q_u (b), Q_i (c)

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7. REFERENCES

Asaeda, T. & Imberger, J (1993). *Structure of bubble plumes in linearly stratified environments*, J. Fluid Mech., 249, pp35-57.

Cooke, G. D., Welch, E. B., Peterson, S. A. & Nichols, S. A (2005). *Restoration and management of lakes and reservoirs*, 3rd ed, CRC, pp591.

Furusato, E., Kunou, M., Niwa, K. & Umeda, M (2015). *Density current flux due to bubble plume in reservoirs, Journal of Japan Society of Civil Engineers*, Ser. G (Environment), 71 (7), Ⅲ, pp455-466. [in Japanese with English abstract]

Furusato, E. & Inuyama, T (2016). *Field observations of intrusion in reservoir induced by bubble plume*, Journal of Japan Society of Civil Engineers, Ser. B1 (Hydraulic Engineering), 72(44), I, pp691-696. [in Japanese with English abstract]

Goossens, L. H. J (1979). *Reservoir destratification with bubble columns*, Thesis, Delft Univ., pp200.

Japan Water Agency (2006). *Water Resources Engineering Department Report*, No. 051404, FY2005, Water quality control by bubble circulation, effective operation technology, March 2006, Incorporated Administrative Agency, Japan Water Agency, General Technology Promotion Office.

Gotou, K., Furusato, E. & Asaeda T (2008). *Study of facility scale for prevention of water bloom by artificial circulation*, Annual Journal of Hydraulic Engineering, JSCE, 52, pp1297-1302. [in Japanese with English abstract]

Kawasaki, H., Kina, T. & Yokomori, M (2006). *Study on a new energy system for dams using compressed air*, ICOLD, 22, No. 19C.

Kranenburg, C (1979). *Destratification of lakes using bubble columns*, Journal of the Hydraulics Division, 4, pp333-1132.

Kobus, H. E (1968). *Analysis of the flow induced by air-bubble systems*, Proc. 11th International Conference on Coastal Engineering, ASCE. London, pp1016-1031.

Lemckert, C. J. & Imberger, J. (1993). *Energetic bubble plumes in arbitrary stratification*. Journal of Hydraulic Engineering, 119, pp680-703.

Matsunashi, S. & Miyanaga, Y. (1990). *A field study on the characteristics of air bubble plume in a reservoir*, J. Hydrosci. Hydraul. Eng., 9, pp65-78.

MacDougall, T. J (1978). *Bubble plumes in stratified environments*, J. Fluid Mech. 85(4), pp655-672.

Milgram, J. H (1983). *Mean flow in round bubble plumes*, J. Fluid Mech., 133, pp345-376.

Ozmidov, R. V (1965). *On the turbulent exchange in a stably stratified ocean*, Atoms. Ocean Phys., 1, pp493-497 (Engl. transl.)

Socolofsky S. A. & Adams E. E (2005). *Role of slip velocity in the behavior of stratified multiphase plumes*, Journal of Hydraulic Engineering, 131(4)

Topham, D. R (1975). *Hydrodynamics of an oil well blowout*, Beaufort Sea Tech, Rep. No. 33.

Umeda, M., Miyazaki, K. & Tomioka, S (2004). *Field measurements of flow field caused by destratification system in a dam reservoir*, Journal of Japan Society of Civil Engineers, No.775/-69, pp55-67. [in Japanese with English abstract]

Umeda, M (2005). *Numerical modeling of desertification by bubble diffusers in a dam reservoir and its validation*, Annual Journal of Hydraulic Engineering, JSCE, 49, pp1165-1170. [in Japanese with English abstract]

Zic, K., Stefan, H. G. & Ellis, C (1992). *Laboratory study of water destratification by a bubble plume*, Journal of Hydraulic Research, 30(1), pp7-27.