ICOLD 75th Annual Meeting
Saint Petersburg, Russia, June 24-29, 2007

Symposium: «Dam Safety Management. Role of State, Private Companies and Public in Designing, Constructing and Operating of Large Dams»

INVESTIGATION OF RELIABILITY ANALYSIS OF EMBANKMENT DAMS CONSIDERING VARIABILITY OF STRENGTH AND DENSITY

Hiroyuki SATOH[1], Yoshikazu YAMAGUCHI[2] and Hitoshi YOSHIDA[3]

INTRODUCTION To meet the public demand to reduce the cost of dam construction, it is necessary to establish a rational design method for embankment dams. As the materials used for embankment dams are of variable strength and density, their design values must be determined based on test results considering variability. Furthermore, the influence of variability of strength and density on the safety of the dam body should be investigated.

According to the present design method for embankment dams in Japan, the design values of material strength and density are decided based on the values of test results considering variability. The design values are generally smaller than the values of the test results, and the difference is sometimes large.

In this study, we first investigate the variability of strength and density from test results in existing embankment dams, and evaluate the difference in design values and test results. Next, based on the results of the first investigation on existing embankment dams, we conduct stability analysis of a model embankment dam using the reliability analysis method considering variability in strength.

We found that the distribution in the test results for strength and density is thought to be a normal distribution, and that the probability of a safety factor less than 1.0 is very small under the present design and construction method.

STRENGTH EVALUATION METHODS USED IN EMBANKMENT DAM DESIGN In Japan, rockfill dam design processes generally allow a generous margin in strength design value to accommodate potential variations in material strength of construction materials. Strength margins in design and construction are decided as follows:

(1) Margin condition 1

Specimens of different types of rock material with different density (i.e., void ratio) are subjected to strength tests, generally triaxial compression tests, and the results are evaluated based on the Mohr-Coulomb failure criterion to determine the cohesion (c) and the internal friction angle (ø) (see (1) in Fig. 1). This is used as the basis for determining the design value, with cohesion of $c = 0$ used as a safety margin (see (2) in Fig. 1).

(2) Margin condition 2

The relationship between density (void ratio) and internal friction angle ø, as determined from the triaxial compression test results, is used to find the density corresponding to the minimum required value of ø. This is called the design value $ø_0$ for the design density or design void ratio (see Fig. 2).

(3) Margin condition 3

Quality control test during construction confirms that the density is above the design density (see Fig. 3).

Figure 4 shows 35 data sets for the discrepancy between the design value $ø_0$ and the laboratory test value ø' for the internal friction angle for rock material at 21 dams (Matsumoto et al., 1982). The number of data sets in the figure is higher than the number of dams because some of the dams have plural rock zones. The design values $ø_0$ in Fig. 4 are on average 2.4° higher than the test values ø', with a maximum of 9.8°. The average of $ø'/ø_0$ is 0.95, while the average of $\tan ø'/\tan ø_0$ is 0.92.

The sections that follow are dedicated to quantitative assessment of the effect of material variability in strength of rock materials on the safety of rockfill dams as described above, based on plane sliding failure probability and Monte Carlo simulation for circle sliding.

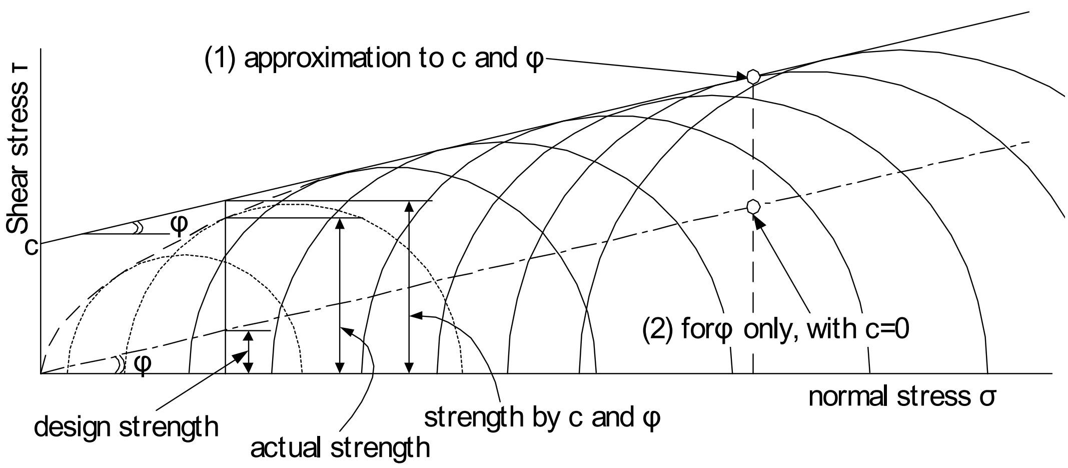


Fig. 1 Design strength and actual strength of rock material – Margin condition 1

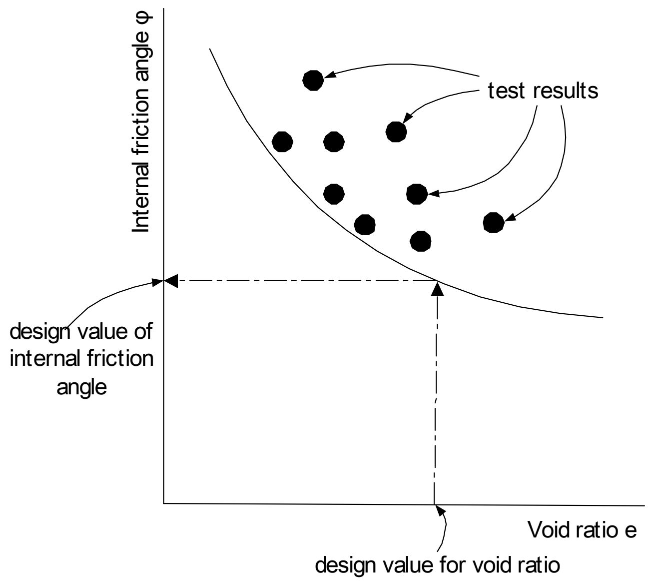


Fig. 2 Typical correlation between void ratio and internal friction angle for rock materials – Margin condition 2

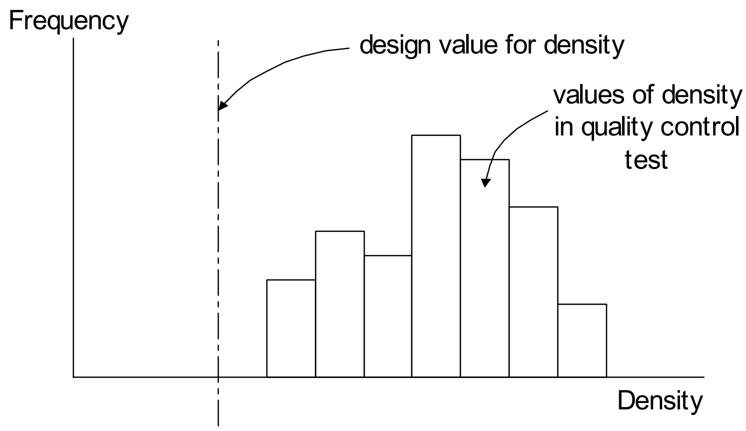


Fig. 3 Example of design value for density of rock material vs. value measured in quality control test during construction – Margin condition 3

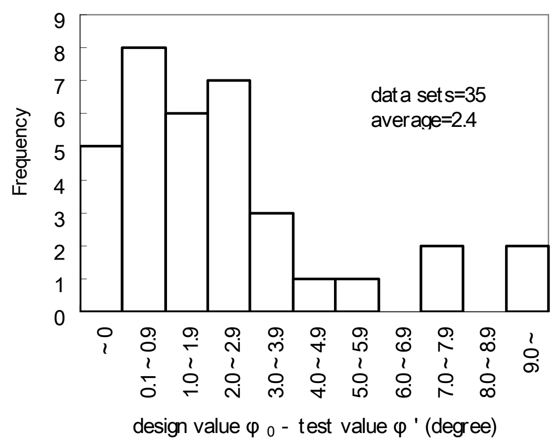


Fig. 4 Internal friction angle for rock material – design values vs. values of laboratory tests (Matsumoto et al., 1982)

FAILURE PROBABILITY OF PLANE SLIDING As seen in the above section, the design strength of the rock material for rockfill dams is based solely on the internal friction angle ø, disregarding cohesion c. For this reason, the minimum safety factor in the stability analysis is determined based on surface slip. This section examines plane sliding failure probability. Circle slip failure probability is discussed in the next section.

Strength and density statistics To calculate the failure probability, the probability distribution and the average and standard deviations of physical properties are needed. In the case of rock material, the number of test results is limited because of the large grain size in actual dams, and because many experimental conditions at the design stage imposes further restrictions. As a result, there is a lack of data on the internal friction angle ø for rock material due to the small number of material and quality control tests, and it is not known whether ø or tanø conforms to the normal distribution.

Figure 5 shows the results of large-scale triaxial compression tests conducted at Dam I, while Fig. 6 shows wet density ρ_t values measured in quality control tests. The limited number of tests in both cases, 9 in Fig. 5 and 14 in Fig. 6, makes it difficult to draw clear conclusions regarding the normal distribution, but the wet density seems to conform to the normal distribution. Since strength and density generally have a good correlation, it can be concluded that the internal friction angle would also conform to the normal distribution.

Table 1 shows statistical data for the internal friction angle (ø) and the coefficient of internal friction angle (tanø) generated by quality control tests on the rock material at Dam I and Dam S, for which the number of samples was comparatively larger than that of other dams. The values in Table 1 indicate that the minimum values of ø and tanø are larger than the corresponding design values, as noted above.

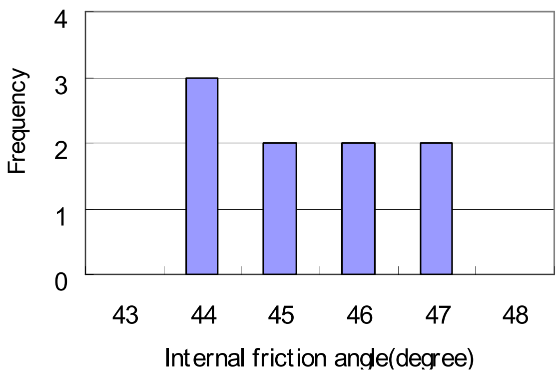


Fig. 5 Example of frequency distribution of internal friction angle ø at Dam I

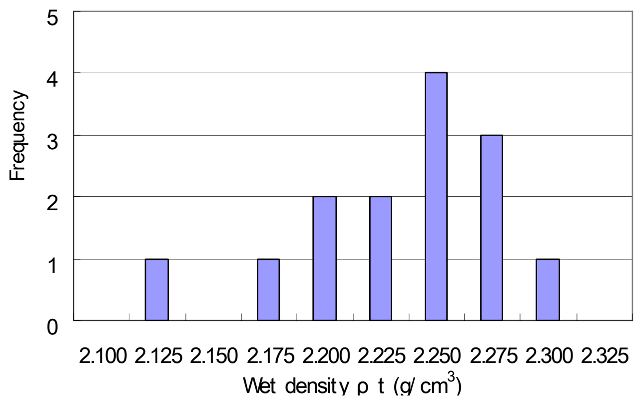


Fig. 6 Example of frequency distribution of wet density ρ_t at Dam I

Table 1 Internal friction angle (ø) and coefficient of internal friction angle (tanø) in quality control tests on rock material at Dam I and Dam S

Dam (data sets)	Physical properties	Average	Standard deviation (coefficient of variation)	Maximum value	Minimum value	Design value
Dam I (9)	Internal friction angle φ (degree)	45.09	1.29 (2.86%)	47.0	43.5	43
	Coefficient of internal friction angle tanφ	1.004	0.0456 (4.54%)	1.072	0.949	0.933
Dam S (7)	Internal friction angle φ (degree)	41.96	1.32 (3.15%)	43.8	40.1	40
	Coefficient of internal friction angle tanφ	0.900	0.0415 (4.61%)	0.959	0.842	0.839

Calculation of failure probability The failure probability can be calculated using a variety of methods, including the first-order second-moment method, Monte Carlo simulation (Hoshiya and Ishii, 1986) and derivation of the probability density function via variable transformation (Itoh and Kameda, 1977). Here, we use the first-order second-moment method. For the purpose of simplification, the failure probability equation assumes one probability variable where only tanø conforms to the normal distribution, because the coefficient of variation of the saturated unit weight γ_{sat} is smaller than the variation of the coefficient of internal friction angle tanø, and also because variation in γ_{sat} has less effect on the failure probability than variation in tanø, as we investigated previously. All other values are constants, and the cohesion of rock material is assumed to be zero.

Resistance force R and slip force S in plane sliding are expressed as follows:

$$R = \left[(\gamma_{sat} - \gamma_w)\cos\theta - k\gamma_{sat}\sin\theta \right]\tan\varphi \tag{1}$$

$$S = (\gamma_{sat} - \gamma_w)\sin\theta + k\gamma_{sat}\cos\theta \tag{2}$$

where

γ_{sat} is the saturated unit weight of rock material,

γ_w is the unit weight of water,

k is the seismic intensity,

θ is the slope gradient, and

ø is the internal friction angle.

The performance function Z is defined as follows:

$$Z = R - S = \left[(\gamma_{sat} - \gamma_w)\cos\theta - k\gamma_{sat}\sin\theta\right]\tan\varphi - \left[(\gamma_{sat} - \gamma_w)\sin\theta + k\gamma_{sat}\cos\theta\right] \tag{3}$$

From the first-order second-moment method, the failure probability P_f is determined by the following equation:

$$P_f = 1 - \Phi(\beta) \tag{4}$$

where

$\beta = \dfrac{\mu_Z}{\sigma_Z}$,

$\mu_Z = A\mu_X - B$ $\sigma_Z = A\sigma_X$,

$A = (\gamma_{sat} - \gamma_w)\cos\theta - k\gamma_{sat}\sin\theta$, and

$B = (\gamma_{sat} - \gamma_w)\sin\theta + k\gamma_{sat}\cos\theta$.

Here, Φ is the standard normal cumulative distribution function in Fig. 7, ß is the safety index, μ_x is the average value of tanø and σ_x is the standard deviation of tanø.

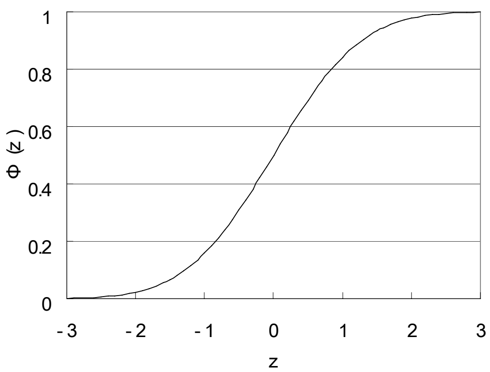


Fig. 7 Standard normal cumulative distribution function

Figure 8 shows how changes in slope gradient and seismic intensity k affect failure probability. Here, the failure probability is calculated using the average and standard deviation values of tanø for Dams I and S shown in Table 1, and the average of γ_{sat} from the quality control tests in Table 2. Because the failure probability for the upstream slope gradient for a seismic intensity of k = 0.15 ranges from 10^{-8} to 10^{-9} for both dams, the plane sliding failure probability is considered sufficiently small. Figure 8 also shows how the failure probability is closely tied to the value of k. This indicates the importance of seismic load evaluation in failure probability calculation.

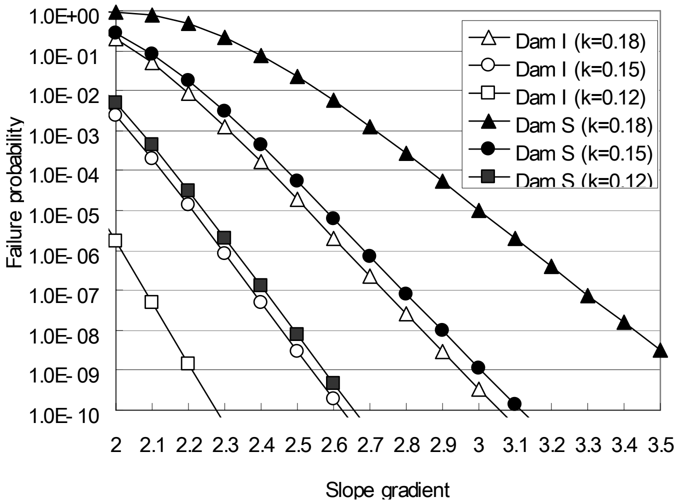


Fig. 8 Failure probability

Table 2 Calculation conditions

Dam	γ sat (kN/m³)	Upstream slope gradient	k
I Dam	23.00	2.5	0.15
S Dam	22.95	2.9	0.15

* The γ_{sat} design values were 20.98 kN/m^3 for Dam I and 22.25 kN/m^3 for Dam S.

When tanø is assumed to conform to the normal distribution, then the failure probability derived from the performance function in Eq. (3) is the same irrespective of the method used, such as the first-order second-moment method, Monte Carlo simulation, or the probability density function based on variable transformation. When ø is assumed to conform to the normal distribution, however, the first-order second-moment method produces a slightly different result compared to the other methods, because if ø conforms to the normal distribution, then tanø does not.

Probability of the safety factor falling below the design safety factor for plane sliding
Based on the values in Table 2, back calculation shows that the internal friction angle ø required for a plane sliding safety factor of 1.0 for impounded water conditions is 36.4° at Dam I and 33.7° at Dam S. If the design values of γ_{sat} are used for the back calculation, the required angle is 37.5° at Dam I and 34.0° at Dam S. The minimum value of ø determined from the quality control tests in Table 1 is 43.5° for Dam I and 40.1° for Dam S, which allows plenty of margin to achieve a safety factor of 1.0. Visual confirmation during construction, which is not quantitative evaluation, ensures that the dam is well built, and it is therefore highly unlikely that the safety factor at an actual rockfill dam would fall below 1.0. For the purpose of confirmation, we conducted a quantitative evaluation of the margin associated with the current design safety factor of 1.2 and calculated the probability of the safety factor falling below 1.2.

$P_{1.2,}$ which is the probability of the actual safety factor falling below 1.2, is calculated from the result of B in Eq. (4) multiplied by 1.2. Figure 9 shows the effect of the slope gradient and seismic intensity k on $P_{1.2}$. Based on the slope gradient and seismic intensity k values shown in Table 2, the $P_{1.2}$ value for both Dams I and S is around 1%. It is assumed that tanø conforms to the normal distribution, but, in reality, the probability of the safety factor falling below the design safety factor of 1.2 is likely to be even less than 1%, since low-strength rock material is unlikely to be used in actual construction as Margin condition 3 described above, while Margin conditions 1 and 2 give considerable margin in the strength setting.

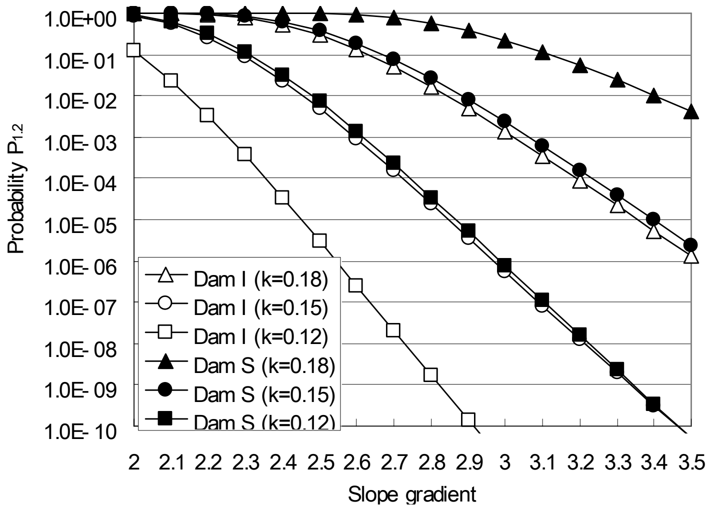


Fig. 9 Probability of the actual safety factor falling below the design safety factor of 1.2

FAILURE PROBABILITY IN CIRCLE SLIDING ANALYSIS

Analysis model Figure 10 shows the analysis model, which has a height of 100 m. The upstream gradient is 1:2.5 and the downstream gradient 1:2.0; these are typical gradients for a rockfill dam in Japan. For simplification purposes, the reservoir water is excluded from the model. The analysis model was divided into elements (see Fig. 11 (a) through (e)) with the internal friction angle ø assigned to each element as a normal random number, and subjected to Monte Carlo simulation for circle sliding analysis. Table 3 shows the analysis conditions, with cohesion c = 0 and horizontal seismic intensity k = 0.15. We investigated three cases using different standard deviations σ for the internal friction angle ø, as shown in Table 3. The number of calculations performed for slip sliding analysis in the Monte Carlo simulation was 10,000 for each case in order to obtain stable statistics values from the analysis results.

Table 3 Analysis conditions

Case #	Unit wet weight (kN/m^3)	Internal friction angle (°)	
		Average μ	Standard deviation σ
Case 1	21.8	45	0.7
Case 2			1.0
Case 3			1.3

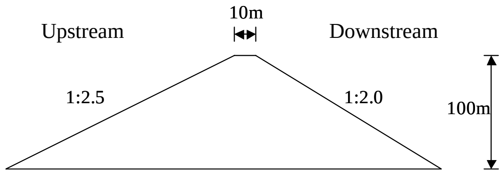


Fig. 10 Analysis model


Fig. 11(a) 1-element model

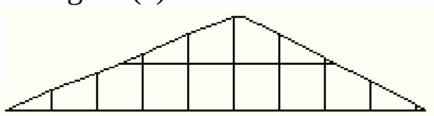


Fig. 11(b) 50-m element model

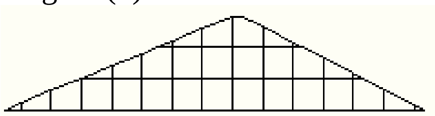


Fig. 11(c) 33-m element model

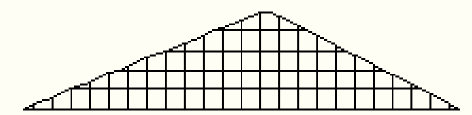


Fig. 11(d) 20-m element model

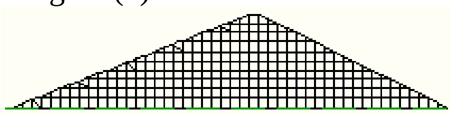
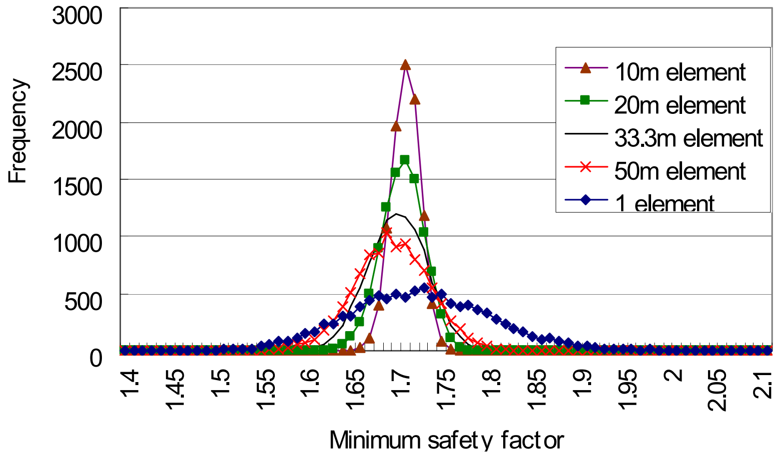


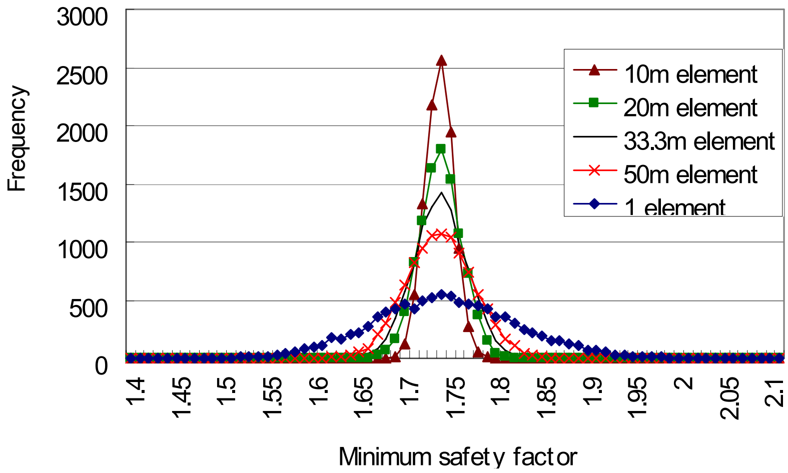
Fig. 11(e) 10-m element model

Effect of slip circle depth from the surface In general, for rock material with cohesion c = 0 in a rockfill dam, the circle that produces the minimum safety factor is the one that cuts through the dam body surface at a shallow depth. This makes it difficult to evaluate the effects of variation in physical properties in the dam body on the safety factor, so we investigated the distribution of the minimum safety factor value relative to the depth from the surface of the circle.

Figure 12(a) shows the frequency distribution of the minimum safety factor where the depth of the circle is at least 10 m from the surface, for Case 3 in Table 3, while Fig. 12(b) shows the corresponding results for a depth of at least 20 m. In both cases, the distribution of the minimum safety factor generally follows the normal distribution. At a depth of at least 10 m, the size of the element affects the average minimum safety factor, but when the depth reaches at least 20 m, the average minimum safety factor is fairly constant irrespective of the element size. Figure 13 shows the distribution of the circles of minimum safety factors when the size of the element is 10 m.



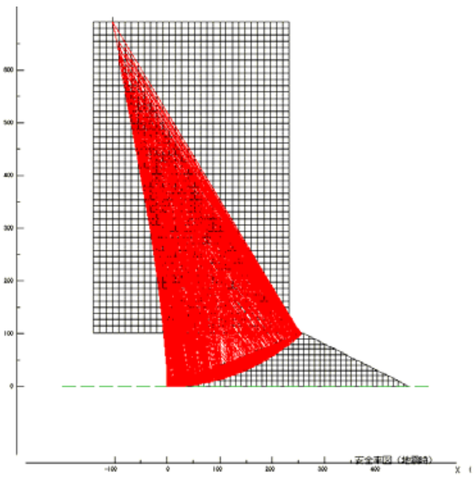
(a) Depth of 10 m or more from surface of dam



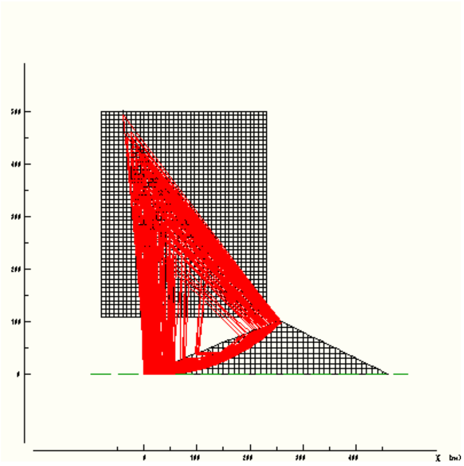
(b) Depth of 20 m or more from surface of dam

Fig. 12 Effect of depth from surface of dam on the minimum safety factor

(for Case 3 in Table 3)



(a) Depth of 10 m or more from surface of dam



(b) Depth of 20 m or more from surface of dam
Fig. 13 Slip circles for minimum safety factor (only 1,000 results)

Effect of element size and standard deviation of internal friction angle Table 4(a) and (b) show the average and standard deviation values for the minimum safety factor at a depth of at least 20 m from the surface of the dam. As with the results in Fig. 12, the average safety factor is fairly constant irrespective of element size and standard deviation of ø. However, the standard deviation of the minimum safety factor is dependent on both element size and standard deviation of ø; the smaller the element size or standard deviation of ø, the smaller the standard deviation of the minimum safety factor.

The comparison of standard deviations for the minimum safety factor in Table 4(b) (Case2/Case1 and Case3/Case1) is similar to the corresponding comparison of standard deviations for ø (1.0/0.7 = 1.429, 1.3/0.7 = 1.857), irrespective of element size. If all other conditions in the analysis are the same, it should be possible to predict the rate of change in the standard deviation of the minimum safety factor relative to the rate of change in the standard deviation of ø.

Table 4(a) Average minimum safety factor

Case #	1 element	50m element	33m element	20m element	10m element
Case 1	1.738	1.738	1.738	1.737	1.737
Case 2	1.739	1.737	1.737	1.737	1.736
Case 3	1.740	1.734	1.734	1.735	1.733

Table 4(b) Standard deviation of minimum safety factor

Case #	1 element	50m element	33m element	20m element	10m element
Case 1	0.042	0.019	0.016	0.012	0.009
Case 2 (Case 2/ Case 1)	0.061 (1.439)	0.028 (1.439)	0.022 (1.431)	0.018 (1.419)	0.012 (1.398)
Case 3 (Case 3/ Case 1)	0.079 (1.890)	0.037 (1.890)	0.029 (1.869)	0.023 (1.839)	0.015 (1.780)

Other relevant considerations The calculation conditions for Case 3 in Table 3 are similar to those of Dam I in Tables 1 and 2. Assuming that a minimum safety factor of 1.733 for the 10-m element size in Case 3 in Table 4(a) is equivalent to the design safety factor of 1.2 (see Fig. 14

(1)), the Case 3 model analysis for a safety factor of 1.0 in the design cross section would produce the following (see Fig. 14 (2) and (3)):

$$1.733 \times 1.0/1.2 = 1.444$$

Figure 13(b) tells us that there is zero probability of the safety factor falling below the minimum of 1.444 in 10,000 calculations in the Monte Carlo simulation. From this, we can assume that the probability of the safety factor in circle sliding analysis falling below 1.0, with a cross-section design based on the average strength value and a required safety factor of 1.2, is actually very small. Additionally, as we saw in the previous sections, there is a further safety margin because of Margin conditions 1 to 3.

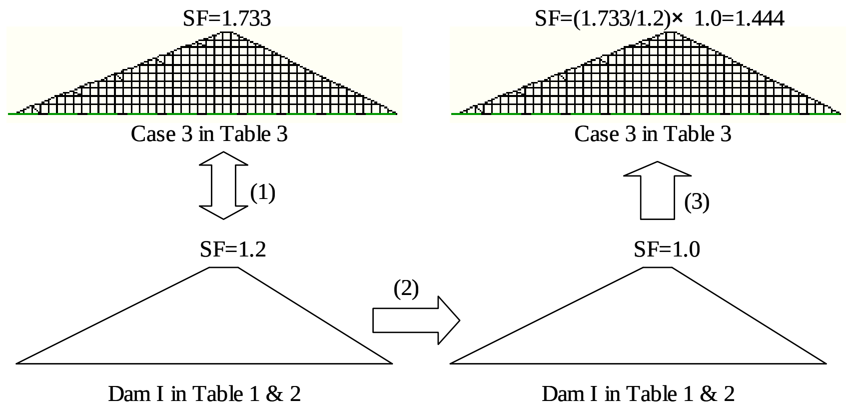


Fig. 14 Explanation of this section

CONCLUSIONS As a basic investigation of performance-based design methods and the reliability analysis method for rockfill dams, we first conducted statistical analysis of strength and density of rock materials of existing rockfill dams. Second, we investigated the influence of strength variability on the safety factor or failure probability of plane sliding or circle sliding. Finally, we estimated the safety margin of rockfill dams constructed based on the present design method. We conclude as follows:

1. In general, the strength of rock materials is decided on the safety side, for example by the method of disregarding the cohesion of rock materials. Although the constraint conditions included a small number of test results for strength and density, the results of statistical analysis revealed that the frequent distribution of strength and density of existing dams follows the normal distribution.

2. Based on the statistical data on strength and density of existing rockfill dams, we applied the reliability analysis method for plane sliding and circle sliding. The first-order second-moment method was used to calculate the failure probability of plane sliding, and we found that the failure probability was very small under the present design method for rockfill dams. To estimate the influence of strength variability on the safety factor of circle sliding, we used Monte-Carlo simulation for slip circle analysis, and we found that the change ratio of the variability of safety factor can be estimated from the change ratio of the variability of strength. The probability that the safety factor of sliding is smaller than 1.0 is estimated as

being very small, if the cross section is designed using the mean value of strength and safety factor of 1.2, and is constructed by the present quality control method.

Investigation of the reliability analysis method should be continued, because the method has several advantages: the influence of variability in physical properties on safety is quantitatively evaluated, and the safety of different structures can be directly compared based on the failure probability.

REFERENCES

Hoshiya, M. and Ishii, K. (1986) *Reliability Analysis Method for Structures,* 208 p. (in Japanese).

Itoh, M. and Kameda, H. (1977) *Probability Concepts in Engineering Planning and Design,* pp. 168-169 (in Japanese).

Matsumoto, T., Watanabe, K. and Yoshinouchi, S. (1982) *Investigation of rock materials of rockfill dams in design and construction,* Technical Note of PWRI, No. 1839, 188 p. (in Japanese).

---

[1] Senior Researcher, Dam Structures Research Team, Hydraulic Engineering Research Group, Public Works Research Institute.

[2] Team Leader, Dam Structures Research Team, Hydraulic Engineering Research Group, Public Works Research Institute.

[3] Group Leader, Hydraulic Engineering Research Group, Public Works Research Institute.