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**APPLICATION OF THE TRANSFER FUNCTION MATRIX METHOD  
IN DAM ENGINEERING\***

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SUMMARY

Transfer function is essential for the seismic design and safety evaluation, etc. of structures. In order to improve the evaluation accuracy of the transfer function, the authors proposed the transfer function matrix method considering mutual interference between vibration directions. In this study, the applicability of the proposed method in the field of dam engineering is investigated. It has been concluded that the proposed method can give out more recognizable dynamic characteristics than the conventional method. It is applicable to the earthquake response prediction of dams at low cost without relying on a numerical model. Furthermore, it shows the possibility for utilizing in the deterioration diagnosis of dams.

RÉSUMÉ

La fonction de transfert est essentielle pour la conception anti-sismique et l'évaluation de la sûreté des ouvrages. Afin d'améliorer la précision de l'évaluation

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\* *Application de la méthode de la fonction/matrice de transfert dans le domaine de l'ingénierie des barrages*

de la fonction de transfert, les auteurs proposent une méthode de fonction/matrice de transfert qui prend en compte l'interférence mutuelle des différents sens des vibrations. Dans cette étude, nous examinons l'applicabilité de la méthode proposée dans le domaine de l'ingénierie des barrages. La conclusion est que la méthode proposée possède des caractéristiques dynamiques plus reconnaissables que celles de la méthode traditionnelle. Elle peut s'appliquer à la prévision de la réaction des barrages aux séismes, à bas coûts et sans besoin de modèle numérique. En outre, elle présente la possibilité d'être utilisée pour le diagnostic des dégâts des barrages.

## 1. INTRODUCTION

Transfer function is one of the indispensable indices for seismic design, seismic safety evaluation, seismic diagnosis, etc. of structures. It is currently estimated by the ratio of the Fourier spectrum of the input wave and the response wave for each direction of vibrations [1]. This procedure is based on the assumption that the response of the structure in a considered direction would be caused by the input vibration only in a corresponding direction by ignoring the mutual interference between the directions of earthquake response. For example, the transfer function of the arch dam as shown in Fig. 1 is estimated by Eq. [1] using the record for each direction of the seismic motion in the foundation and the dam crest.

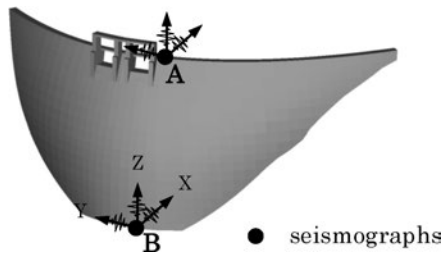


Fig. 1

Schematic view of dam response due to earthquakes  
*Vue schématique de la réaction du barrage aux séismes*

$$T_i = \frac{S_i^A}{S_i^B} \quad (i = X, Y, Z) \quad [1]$$

Where,  $S$  is the Fourier spectrum of the seismic acceleration record, and  $T$  is the transfer function.  $A$  and  $B$  are subscripts indicating crest and foundation respectively,  $X, Y, Z$  indicates the direction of vibration component.

The seismic response in the specific direction in the crest also includes the influence of the vibration components in the other directions on the foundation. For this reason, it is considered that the mutual interference between the directions varies according to the characteristics of seismic motions. In the method of

Eq. [1] (hereinafter referred to as the conventional method), mutual interference between vibration directions are ignored. It is assumed that the seismic motions in the three directions and the earthquake responses at the crest occur independently in each direction. As a result, the transfer functions calculated by Eq. [1] could show large variations in these shapes for different earthquakes. It may be difficult to read the predominant frequency of the transfer function which identify the natural frequency and the dynamic characteristics of the structure.

Focusing on the mutual interference between vibration directions, the authors have proposed a practical method for the estimation of the transfer functions which are decomposed into three components corresponding to the vibrating direction and contribution corresponding to the other two orthogonal directions, respectively. The details of this method are given in the relevant reference [2]. In order to consider the mutual interference between the vibrating directions of the input and the responses, the transfer function matrix shown in Eq. [2] is calculated using three sets of seismic records. The dynamic characteristics of the structure can be more clearly identified by the nine components of the transfer function.

$$\begin{Bmatrix} S_X^A \\ S_Y^A \\ S_Z^A \end{Bmatrix} = \begin{bmatrix} T_{XX} & T_{XY} & T_{XZ} \\ T_{YX} & T_{YY} & T_{YZ} \\ T_{ZX} & T_{ZY} & T_{ZZ} \end{bmatrix} \begin{Bmatrix} S_X^B \\ S_Y^B \\ S_Z^B \end{Bmatrix} \quad [2]$$

$T_{ij}$  ( $i = X, Y, Z$ ) in Eq. [2] are the transfer functions showing the response characteristic in the direction  $i$  at the point  $A$  with respect to the seismic motion in the same direction of the point  $B$  in Fig. 1, which corresponds to the transfer function by the conventional method (Eq. [1]). Each component  $T_{ij}$  ( $i, j = X, Y, Z$ , while,  $i \neq j$ ) is a newly introduced contribution transfer function to take mutual interference between the directions into account. For example,  $T_{XY}$  is the response characteristic of the direction  $X$  at the point  $A$  with respect to the vibration in the direction  $Y$  at the point  $B$ . And it should be noticed that each variable of Eq. [2] is a function of frequency. Hereinafter the method is referred to as the "proposed method" in this paper.

In this paper, the applicability of the proposed method in the field of dam engineering is discussed. It is examined that the predominant frequencies of dams are evaluated with higher accuracy, and the prediction of the earthquake response of the structures is conducted at low cost without relying on numerical analysis model. The attempt to detect the deterioration and/or the damages of dams by evaluating transfer function matrix is also mentioned.

## 2. EVALUATION OF THE DYNAMIC CHARACTERISTICS OF DAMS

In this section, as an application of the proposed method, the evaluation of the dynamic characteristics of an existing dam is described. The predominant frequency of a concrete gravity dam of 145 m high and 462 m long at the crest

is investigated using the earthquake records monitored in the dam. The locations of the seismographs are shown in Fig. 2 in a downstream elevation. Each seismograph has three components in the stream direction, the cross stream direction and the vertical direction. About 480 earthquake records have been obtained from 1994 so far. As will be described later, the earthquake response of the dam shows a slightly large variation in the predominant frequency evaluated by the conventional method. But no trend over time is recognized in these variations. The linear relationship in the response accelerations of the foundation and the crest are generally found in all monitored earthquakes [3]. Therefore, it is thought that the earthquake responses of the dam are confined in the linear state and its dynamic characteristics are maintained stable throughout the seismic monitoring period.

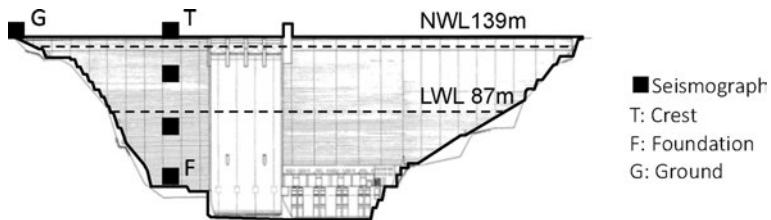


Fig. 2  
Arrangement of earthquake monitoring devices  
*Mise en place des dispositifs de surveillance des séismes*

The earthquake records for the evaluation of the dynamic characteristics by the proposed method are examined in view of the reservoir water depths and the amplitude of the acceleration response at the crest (T in Fig. 2)) in the stream direction. Finally 10 sets of the earthquake records shown in Table 1 are selected. 8 sets and other 2 sets correspond to the states close to the high water level and, the medial water level and the low water level, respectively. The water depths of each set are 0.89 H, 0.38 H and 0.07 H (H is the high water depth), respectively. Earthquake records at the high water level condition were obtained during The Mid Niigata prefecture Earthquake in 2004 (M 6.8). The number of data used in this study is 30, which is about 6% of the total number of the earthquake records monitored at the dam.

The transfer functions between the foundation (seismograph F in Fig. 2) and the crest (seismograph T in Fig. 2) in the stream direction are evaluated by the proposed method as shown in Fig. 3. The predominant frequencies (indicated by  $\downarrow$  in Fig. 3) are listed in Table 2. These results are compared with the predominant frequencies estimated by the conventional method in Fig. 4. The regression line (solid line) in the Fig. 4 is based on reference [4] and for ones by the conventional method. It is a regression with the dimensionless water level ( $h$  (depth of water) /  $H$  (depth relative to full water level)) and temperature. It is considered as the average value of the predominant frequency estimated by the conventional method.

The following discussion will focus on the transfer function obtained by the proposed method and the estimation accuracy of the predominant frequency of the

dam identified as the frequency peaks of the transfer function. Regardless of the amplitude variation of the acceleration response of the dam (0.17 – 6.00 m/s<sup>2</sup>), the proposed method seems to provide relatively identical transfer functions (See Fig. 3), resulting clear identification of the predominant frequency as the lower peaks. While the lower peaks are in a substantially constant range around 3.1 Hz or 3.2 Hz in the high water level condition (No. 1 to No. 8 in Table 1), these slightly shift to the higher frequency according to the water level in the medial water level and the low water level conditions. Furthermore, the transfer function by the proposed method contributes to the improvement of identification accuracy of higher predominant frequencies. For example, the second order predominant frequency in the stream direction is recognized to be 4.1Hz in Fig. 3, which is confirmed with the result of eigenvalue analysis, not involved in this paper.

Table 1  
Earthquake records for evaluation of transfer function matrices

NO.	DATE AND TIME OF OCCURRENCE	MAX. ACCELERATION AT THE CREST (M/S <sup>2</sup> )	WATER DEPTH
1	2004/10/27 10:40:56	6.00	136.2m (High water level)
	2004/10/23 18:34:12	4.98	
	2004/10/23 17:56:08	4.55	
2	2004/10/23 18:03:11	3.36	
	2004/10/25 06:05:04	2.77	
	2004/11/08 11:16:04	1.40	
3	2004/10/23 18:11:30	1.69	
	2004/11/04 08:57:37	1.53	
	2004/11/08 11:16:04	1.40	
4	2004/10/23 19:46:05	1.17	
	2004/10/25 00:28:17	0.94	
	2004/10/23 21:44:34	0.73	
5	2004/10/23 18:07:39	0.66	
	2004/11/08 11:32:22	0.65	
	2004/11/10 03:43:14	0.62	
6	2004/11/08 11:32:22	0.65	
	2004/11/10 03:43:14	0.62	
	2004/10/24 09:28:12	0.50	
7	2004/10/23 19:36:54	0.48	
	2004/11/08 11:27:16	0.42	
	2004/11/09 04:16:06	0.41	
8	2004/11/08 11:27:16	0.42	
	2004/11/09 04:16:06	0.41	
	2004/11/06 02:53:27	0.40	
9	2005/01/18 21:50:33	0.37	118.4m (Medial level)
	2010/05/01 18:20:37	0.67	
	1998/02/21 09:55:42	0.44	
10	2011/03/11 14:54:42	0.17	88.7m (Low level)
	2011/03/11 15:15:34	0.18	
	2011/03/19 18:56:48	0.17	

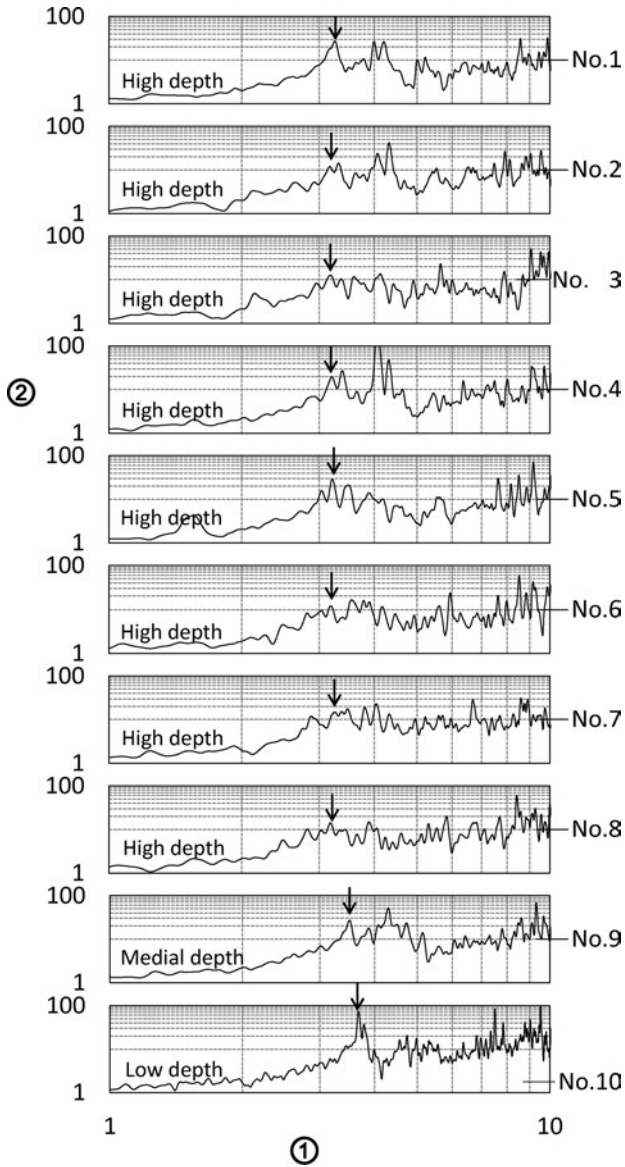


Fig. 3

Predominant frequency of the dam by transfer function matrix  
 Fréquence prédominante du barrage avec la fonction/matrice de transfert

- |                       |                        |
|-----------------------|------------------------|
| ① Frequency (Hz)      | ① Fréquence            |
| ② Amplification ratio | ② Taux d'amplification |



## C. 4

While the predominant frequencies by the proposed method are identical under the high water depth condition and seems to fit the regression line, ones by the conventional method vary in the range of about 0.5 Hz even in the almost same water level. It means that the evaluation accuracy of the proposed method is higher than the conventional method. The higher accuracy of the evaluation of the transfer function and the predominant frequencies attribute to the consideration in the proposed method on mutual interference of the dam responses in each direction. It is known that the predominant frequency of the concrete gravity dam shifts to the higher frequency with the declination of the water depth due to the dynamic interaction between the dam and the water [3], [4]. Such tendency is clearly confirmed in the evaluation of the proposed method.

As described above, the evaluation of the transfer function by the proposed method is relatively reasonable and accurate compared with the conventional method. It contributes to the improvement of the identification of the higher order predominant frequencies also with higher accuracy. It is considered to be effective and applicable for evaluating the dynamic characteristics of structures such as dams.

### 3. PREDICTION OF THE EARTHQUAKE RESPONSE OF DAMS

#### 3.1. METHOD

The response analysis method using the numerical model or that obtains the maximum response of a dam by the statistical treatment using the response spectrum of the specified ground motion [5] are common for predicting the earthquake response of a dam. Similar to the response spectrum method, the earthquake response of a structure can be predicted based on the transfer function [1]. In the case of a dam which are a large scale and prevail 3-D characteristics, it is difficult to apply such methods. No practical application examples can be referred. This is certainly because it is difficult to accurately evaluate the transfer function of the dam. Here the application of the proposed method is described for the prediction of the earthquake response of a dam.

The earthquake response prediction of a dam by applying the proposed method can be carried out in the following three steps. These are (1) to find the transfer function matrix by the proposed method based on the past earthquake records, (2) to find the Fourier spectrum of the response of the dam as the product of the Fourier spectrum of the objective seismic motion and the transfer function matrix, and (3) finally to convert the Fourier spectrum to the time domain



to obtain the realistic earthquake response of the dam. This process is shown in Fig. 5 as a flowchart. Hereinafter, this will be concretely illustrated. The prediction results and the real earthquake records will be compared.

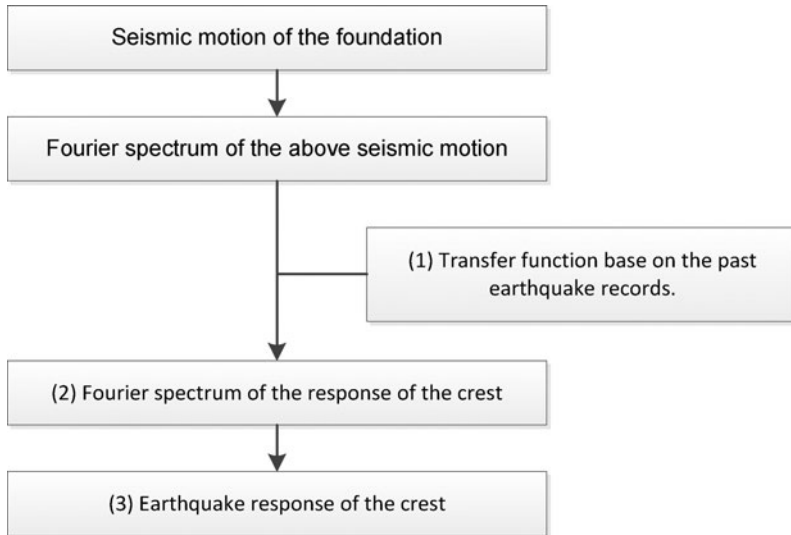


Fig. 5

Flowchart of earthquake response estimation of dams  
using transfer function matrix

*Diagramme de l'estimation de la réaction des barrages aux séismes utilisant  
la fonction/matrice de transfert*

### 3.2. CONDITIONS

An existing concrete gravity dam shown in Fig. 2 is selected as the application object. The responses to the aftershocks of The Mid Niigata prefecture Earthquake in 2004, which caused the largest response to this dam (October 27, 2004, M 6.1, epicenter distance 23 km) is predicted by the proposed method. Fig. 6 shows the acceleration records of the said earthquake monitored at the bottom gallery (seismograph F in Fig. 3). The transfer function matrix is evaluated using three smaller earthquake records (data set No. 7 shown in Table 1) which occurred at almost the same water level so that the influence of the reservoir water can be similarly taken into consideration.

C.4

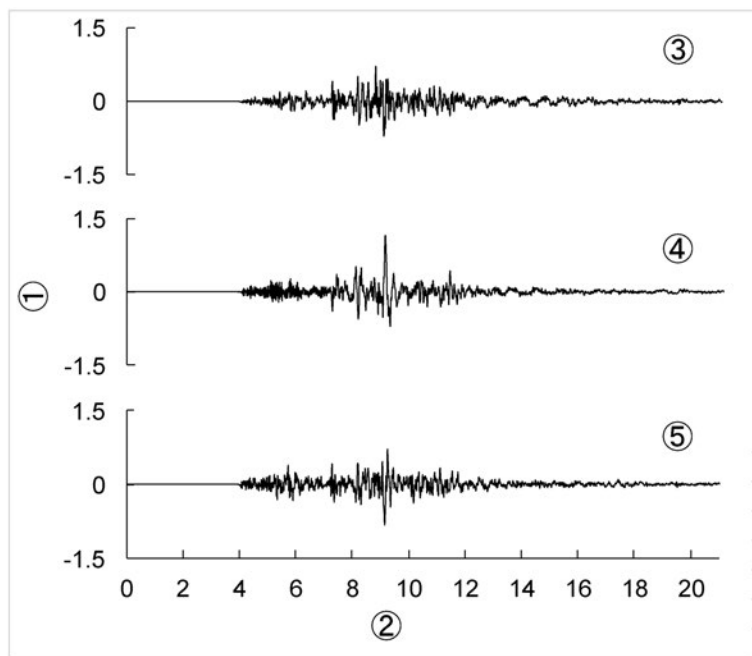


Fig. 6  
 Monitored acceleration at the dam bottom  
*Accélération surveillée au fond du barrage*

- |                                    |   |
|------------------------------------|---|
| ① Acceleration (m/s <sup>2</sup> ) | ① <i>Accélération (m/s<sup>2</sup>)</i> |
| ② Time (sec.)                      | ② <i>Temps (seconde)</i>                |
| ③ Stream direction                 | ③ <i>Sens du courant</i>                |
| ④ Cross stream direction           | ④ <i>Direction longitudinale</i>        |
| ⑤ Vertical direction               | ⑤ <i>Sens vertical</i>                  |

3.3. RESULTS

The predicted acceleration responses in three directions at the crest of the dam are shown in Fig. 7 comparing to the monitored earthquake responses. The waveforms of the major motion and the maximum acceleration values are generally consistent with each other. It is thought that the accuracy of the prediction with the proposed method is not inferior to that of the analysis using numerical models. The applicability of the proposed method is clear to predict the response of the dam. In addition, the method shown here is excellent characterized by that the mechanical properties of the dam are inherently reflected in the process of the evaluation of the transfer function matrix. This fact excludes the uncertainty concerning the analysis parameters used in the response analysis for the numerical models.

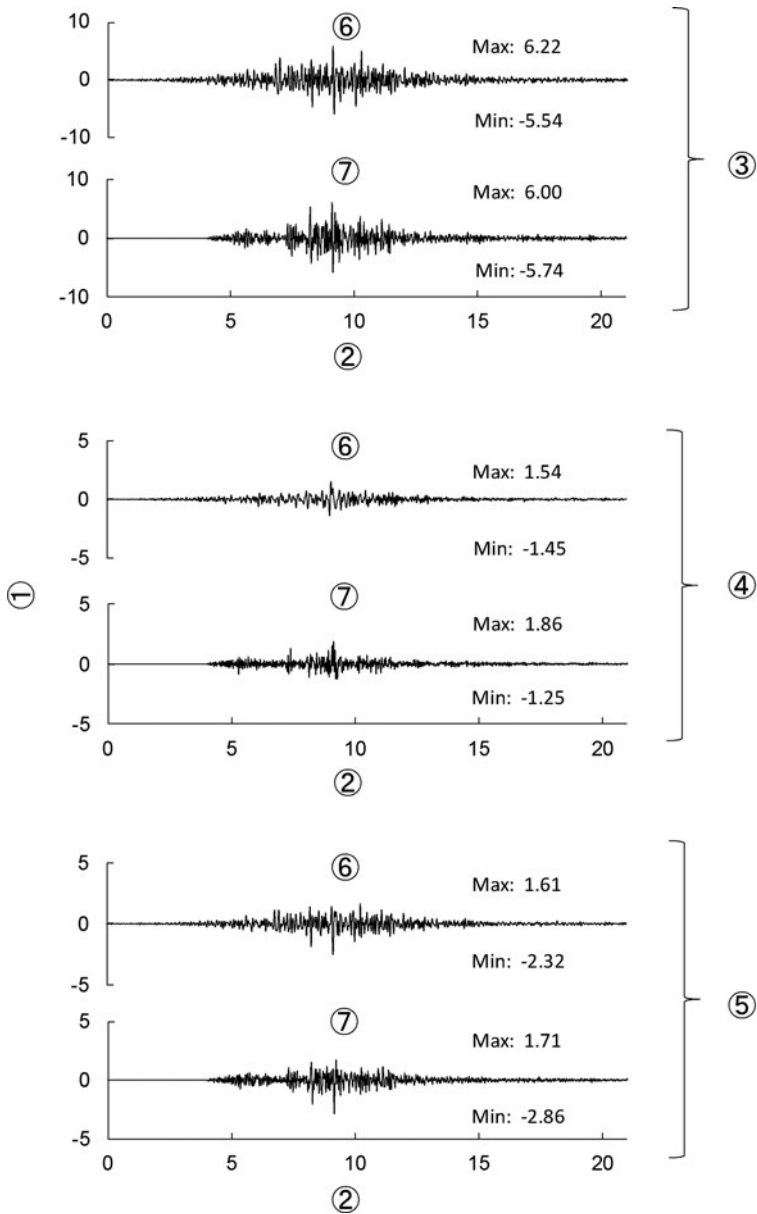


Fig. 7  
 Comparison of dam response  
*Comparaison de la réaction de barrages*

- |                            |  |
|----------------------------|--|
| ① Acceleration ( $m/s^2$ ) | ① <i>Accélération (<math>m/s^2</math>)</i> |
| ② Time (sec.)              | ② <i>Temps (seconde)</i>                   |
| ③ Stream direction         | ③ <i>Sens du courant</i>                   |
| ④ Cross stream direction   | ④ <i>Direction longitudinale</i>           |
| ⑤ Vertical direction       | ⑤ <i>Sens vertical</i>                     |
| ⑥ Analysis                 | ⑥ <i>Une analyse</i>                       |
| ⑦ Earthquake record        | ⑦ <i>Records de tremblement de terre</i>   |

#### 4. DETERIORATION DIAGNOSIS OF DAMS

Concrete dams may deteriorate in the surface layer due to temperature variation, chemical reaction / corrosion, repeated earthquake striking, and so on. The related references [8], [9] pointed out that the elastic modulus of the surface layer concrete structures exposed to the natural environment for 40 years may reduce about 10%. In these cases it is mandatory to clarify the deterioration situation and if necessary, to provide corresponding measures for the maintenance, disaster prevention and effective utilization of the dam.

An investigation on the variation of the dynamic characteristics of the dam by analyzing the earthquake records is conceivable as a method of detecting the deterioration situation of a dam. Sasaki [6] proposed a method to estimate the damage status of a dam due to an earthquake or other loads, using an index of the dam's predominant frequency. One of the authors examined the historical variation of the transfer function of a dam to evaluate the damage due to the large earthquake impact [3]. However, in these studies, there is a problem in the evaluation accuracy of the transfer function. Here, the applicability of the proposed method is discussed in the deterioration diagnosis of dams.

Assuming that the surface of the dam concrete is deteriorated, the transfer function matrix of such status is compared with that of the original sound status in this study. To illustrate the applicability, the transfer function by the conventional method is compared. As shown in Fig. 8, it is assumed that the surface layer of the dam concrete in 1.0 m deep is deteriorated, resulting that its elastic modulus decreased by 10% (The original elastic modulus was 26500 N/mm<sup>2</sup>). The transfer function is examined using the hypothetical responses of the dam. The dam responses are calculated by the dynamic analyses method using three sets of earthquake inputs of an existing dam. The program UNIVERS [7] is used for the earthquake response analyses. The stream direction component of the transfer function is focused for the investigation of the deterioration. The results are shown in Fig. 9.

Since the transfer function matrix exhibits smooth fluctuation, the influence of the assumed deterioration is recognized as the slight change in the transfer function. The predominant frequency decreases only about 0.05 Hz. On the other hand, the transfer function by the conventional method is difficult to read the peak value due to the overlaying a sharp and small fluctuation over the wide frequency range. The difference in predominant frequency due to the assumed deterioration is not clear. In practice, it might be difficult to detect such kind of deterioration as assume here in the change of the transfer functions in both methods. However, it is presumed that the dynamic characteristics will fluctuate obviously in the case where a strong earthquake might cause a wide range of cracks in a dam. Therefore, the high-accuracy transfer function evaluation by the proposed method is expected to be an effective way of detecting deterioration. This subject should be further studied in the next step.

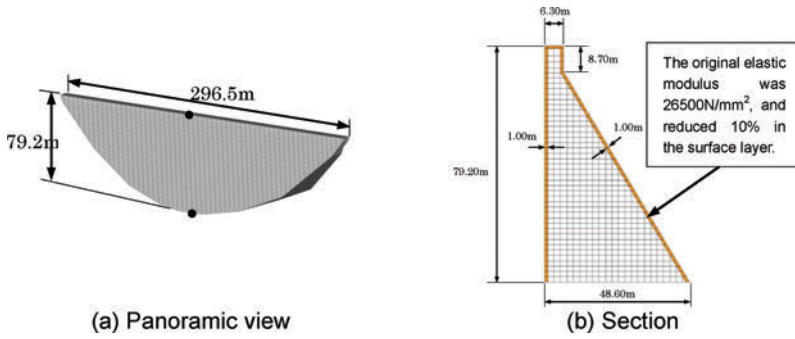


Fig. 8  
 Numerical model for deterioration diagnosis  
 Modèle numérique pour diagnostic des dégâts

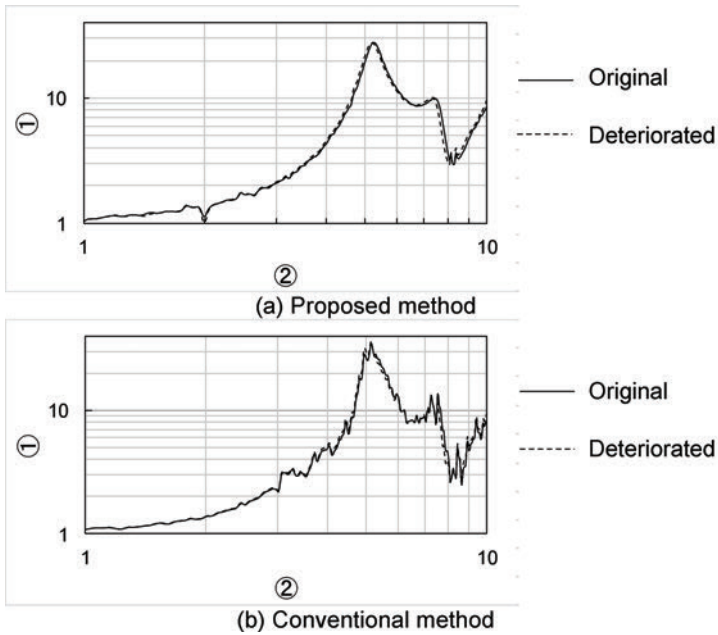


Fig. 9  
 Monitored acceleration at the dam bottom  
 Accélération surveillée au fond du barrage

- |                  |                        |
|------------------|------------------------|
| ① Amplification  | ① Taux d'amplification |
| ② Frequency (Hz) | ② Fréquence            |

## 5. CONCLUSION

In this paper, the applicability of the transfer function matrix method in dam engineering has been investigated. The following conclusions are obtained.

1) In the transfer function matrix method, the mutual interference between the directions of the inputs and the responses is considered. As a result, the transfer function matrix can quantify the dynamic characteristics of dams more clearly than in the conventional method.

2) Earthquake response prediction by applying the transfer function matrix method has been confirmed to reproduce the actual response of the dam with good accuracy. In this application, the method is excellent characterized by that the dynamic characteristics of the dam is reflected at the time in the process of the evaluation of the transfer function matrix. It excludes the uncertainties resulting from the numerical model representation and the dynamic parameters setting which are usually needed in the common earthquake response analysis.

3) Transfer function matrix method may be applied in the deterioration diagnosis of dams. Compared with the conventional method, it can provide a relative clear recognition on the variation of the predominant frequencies of dams. However, in practice it might be difficult to detect such slight deterioration of the surface layer of dams. The further study is necessary on this issue.

## REFERENCES

- [1] OSAKI Y. Introduction to spectral analysis of earthquake ground motion, *Kajima Press Conference*, 1994 (in Japanese)
- [2] CAO Z., KASHIWAYANAGI M., YOSHIDA M., ASAKA H. An high-accuracy evaluation method of transfer function considering the interferences between vibrating directions and its application in dam engineering *Electric Power Civil Engineering*, No.390 pp.13-21 July, 2017 (in Japanese)
- [3] KASHIWAYANAGI M., OHNISHI T., NAGATA N., HAYAKAWA S. Evaluation of dynamic characteristics of concrete dams based on earthquake records, *Proceedings of the 44<sup>th</sup> Symposium on Rock Mechanics*, pp.166-171, 2016 (in Japanese)
- [4] KONDO M., KOBORI T., KASIMA T., SASAKI T. Variation of the fundamental frequency of concrete gravity dams and multiple regression analysis, *Dam Engineering*, 25 (1), pp. 16-28, 2015 (in Japanese)

- [5] NAGAYAMA I., SASAKI T. Earthquake response analysis of arch dams by response spectrum method, *Civil Engineering Journal*, 35-5, pp.22-29, 1993 (in Japanese)
- [6] SASAKI T., KANANAWA K., YAMAGUCHI Y. A study on the estimation method of damage of concrete gravity dams induced by earthquakes, *The 58th Annual Meeting of JSCE*, I-195, 2003 (in Japanese)
- [7] ARIGA Y., CAO Z., WATANABE H. Study on 3-D dynamic analysis of arch dam against strong earthquake motion considering discontinuous behavior of joints, *Journal of JSCE*, No. 759 / I-67, pp.53-67, 2004.04 (in Japanese)
- [8] EGAWA K., CHIYODA M., NOMOTO T., NOGUCHI H. Basic research on frost harmfulness of dam concrete based on long-term exposure test, *Journal of JSCE*, No. 472 / V-20, pp. 39- 48, 1993 (in Japanese)
- [9] NARUOKA M., HARAT., TAKEISHI S. Evaluation of freezing dissolution resistance of dam concrete and the influence of surface deterioration, *Annual Meeting of JSCE*, Vol.29, No.2, 2007 (in Japanese)